Design of a Two-Level Adaptive Flight Control System

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A design of a two-level adaptive flight controller for a STOL aircraft with unknown dynamics is described. The approach appears to overcome some of the limitations that are inherent in the design of linear optimal and conventional adaptive controllers. In particular, it is assumed that the state variables are not accessible. For estimating the state variables an adaptive observer is developed which has an exponential rate of convergence, and which simultaneously models the dynamics of the unknown plant. Control at the first level is provided by an updated optimal controller, while that at the second level is provided by an error servo. Some examples of simulation studies that were carried out for the pitch attitude control system for two different conditions are given.

Introduction

MUCH work has been done during the past few years on the design of controllers which can simultaneously satisfy the desired response characteristics of a system and cope with the modeling inaccuracies and environmental uncertainties. Typical examples of such works are found in the publications by Landau and Courtiol,[1] Horowitz et al.,[2] and Ohta and Sugiura.[3]

Of particular interest here is the work done on two-level controllers.[4,5] A common characteristic of these systems is that each controller operates in one of two modes and each plant is controlled by both feedback and feedforward. Basically, the first-level controller can produce the desired response characteristics if the plant is known and deterministic. If, however, the plant is unknown and nondeterministic, the second level compensates for modeling inaccuracies and uncertainties. More specifically, the system described by Preusche[4] consists of an optimal controller with model state feedback at the first level and an error servo at the second level. By assuming that the deviations between the dynamic behaviors of the plant and the model are small, the problems of state measurement and sensitivity, which are characteristic of optimal controllers, were avoided by Preusche. In the two-level adaptive controller reported by Nikiforuk et al.[5] a "characteristic vector," which is an implicit function of the unknown environment of the plant, was used. This adaptive controller was designed for multi-input multi-output unknown plants and employed a Liapunov-type signal synthesis procedure. This design procedure was subsequently applied to a gust alleviation control system.[6]

In this paper a two-level adaptive controller is developed for applications to aircraft-type systems – systems which experience substantial changes in their dynamic characteristics. Previous work in this area includes that done by Narendra and Tripathi,[7] who studied the identification and optimization of helicopter dynamics assuming the accessibility of the state variables. To avoid the use of state variables, an adaptive observer is used in this paper. Such observers, as has been shown by Carroll and Lindorff,[8] Kudva and Narendra,[9] Kreisselmeier,[10] and Nikiforuk et al.,[11] can be used in both single-input single-output and multi-input multi-output systems for providing estimates of the state variables and the parameters of an unknown plant using only input-output data. In the system to be described, an adaptive observer is used explicitly in the first level for the purpose of generating the signals needed for updating.

In the following paragraphs, first a description is given of the overall two-level adaptive control scheme that was adopted. This is followed by the development of an adaptive observer with an exponential rate of convergence. This observer, which is to be used for state estimation and plant indentification, is developed using an equiobservable canonical form previously described by the authors[12] and the least-squares method.[13] The optimal controller is then described. Finally, some simulation results are presented which were obtained from a study that was made of the controller's application to the pitch attitude control of a STOL.

Configuration of the Two-Level Adaptive Control Scheme

The purpose of the two-level adaptive control system that is proposed here is to control the outputs of a real but unknown plant so as to minimize a given performance criterion. This system, which is illustrated in Fig. 1, consists of four main blocks: 1) a real plant with unknown dynamics, 2) an adaptively identified model of the plant (the adaptive observer), 3) an updated optimal controller with model state variable feedback, and 4) an error servo. This system also contains two control switches. The first-level control becomes operative when switch 1 is closed and the second-level control becomes operative when switch 2 is closed. The estimation and the identification of the unknown plant are carried out by

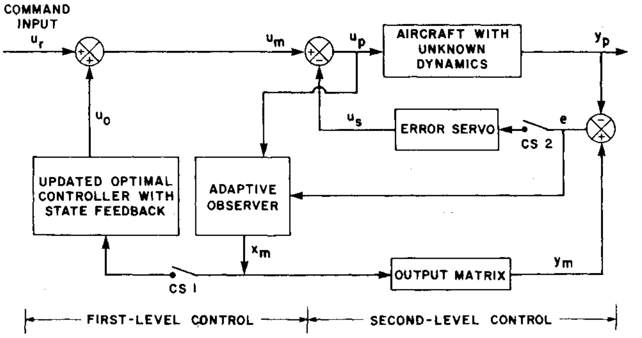

Fig. 1 Configuration of the two-level adaptive control system.

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the adaptive observer regardless of the position of these switches.

This system operates as follows. All known properties (structure and parameters) of the plant and the command inputs are handled by the first-level control, which contains the adaptive observer and an updated optimal controller. These two elements are updated periodically, using the current estimate of the plant dynamics, and provide a first-level compensation. As a consequence, at the second level the error servo has to compensate only for the small deviations which exist between the model and the plant and which are due to the unidentified or unregarded properties of the plant.

Some of the advantages which are associated with this system are as follows:

1) The optimal controller uses estimated state variable feedback and generates an optimal policy that is based on the current knowledge of the unknown plant.

2) The adaptive observer considers only the essential dynamic properties of the plant. Thus, it may be of lower order and simpler than the plant itself.

3) A conventional controller may be used as an error servo system. Should the first-level control fail, the conventional control system may be used as a backup system.

Design of the Adaptive Observer

Consider a linear time-invariant dynamical plant described by the equation

$$\dot{x}(t) = Fx(t) + gu_p(t) \tag{1a}$$

$$y_p(t) = hx(t) \tag{1b}$$

where $x(t) \in R^n$ is a state vector and $u_p(t)$ is an input and $y_p(t)$ is an output. The unknown triple (F,g,h) is of appropriate size and is assumed to be completely controllable and observable. It is also assumed that the plant is free of disturbances and that the only accessible signals are $u_p(t)$ and $y_p(t)$, which are noise free.

A specific canonical form is selected for the design of an adaptive model of the plant. It is shown in the Appendix that if the plant described by Eq. (1) is completely observable, the pair (F,h) can be represented by

$$F = \Lambda + [f:\ldots:f] \qquad h = [1:\ldots:1] \tag{2}$$

where $\Lambda \in R^{n\times n}$ is a diagonal matrix with arbitrary but known constants and negative elements $-\lambda_i$ ($i=1,\ldots,n$; and $\lambda_i \neq \lambda_j$ for $i \neq j$). The f and $g \in R^n$ are unknown vectors whose parameters are to be identified. Only the single-input single-output case is considered in this study. The multivariable case can be handled using the multivariable version of the canonical form discussed in Ref. 12.

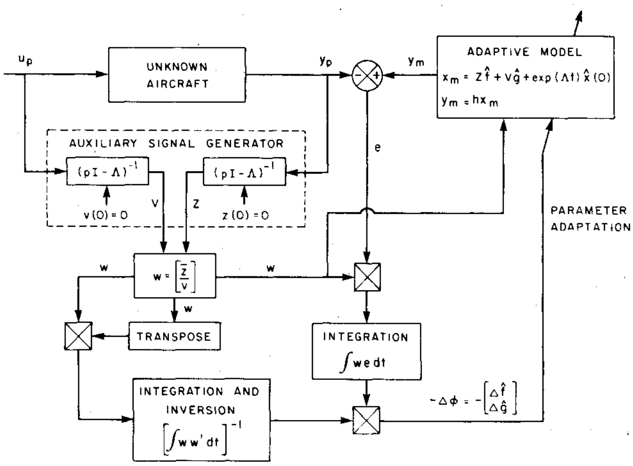

Fig. 2 Schematic diagram of the adaptive observer.

Let the control switches be in the off position in Fig. 1 (that is $u_p = u_m = u_r$). In addition, let the adaptive model of the plant be

$$\dot{x}_m(t) = \hat{F}x_m(t) + \hat{g}u_p(t) \tag{3a}$$

$$y_m(t) = hx_m(t) \tag{3b}$$

where $x_m(t) \in R^n$ and $\hat{F}$ has the same form as F in Eq. (2), and $\hat{f}$ and $\hat{g}$, the estimated values of f and g, which are estimated as follows.

Using the measurable signals $y_p(t)$ and $u_p(t)$, let the auxiliary signals, $Z(t)$ and $V(t) \in R^{n\times n}$, and $\bar{Z}(t)$ and $\bar{V}(t) \in R^n$, be defined as

$$Z(t) = [pI - \Lambda]^{-1} y_p(t) \tag{4a}$$

$$\bar{Z}(t) = Z(t)h' \tag{4b}$$

$$V(t) = [pI - \Lambda]^{-1} u_p(t) \tag{4c}$$

$$\bar{V}(t) = V(t)h' \tag{4d}$$

where $p \triangleq \mathrm{d}/\mathrm{d}t$ is the differential operator, and $[\cdot]'$ is a transpose. A filter whose eigenvalues are at the designer's discretion is introduced in Eq. (4). This filter, as illustrated in Fig. 2, is driven by either y_p or u_p of the plant, and generates the auxiliary signals $Z(t)$ and $V(t)$. These signals are available and are used for representing the plant dynamics as an algebraic function of them. Without loss of generality, the initial values of the filters, $Z(0)$ and $V(0)$, are set equal to zero. Then Eq. (1) can be expressed as

$$x(t) = Z(t)f + V(t)g + \exp[\Lambda t]\cdot x(0) \tag{5a}$$

$$y_p(t) = \bar{Z}'(t)f + \bar{V}'(t)g + h\exp[\Lambda t]\cdot x(0) \tag{5b}$$

Let the parameter vector to be identified, $\phi \in R^{2n}$, and the signal vector, $w(t) \in R^{2n}$, be defined as

$$\phi = \begin{bmatrix} f \\ g \end{bmatrix} \qquad w(t) = \begin{bmatrix} \bar{Z}(t) \\ \bar{V}(t) \end{bmatrix} \tag{6}$$

Hence, Eq. (5b) can be rewritten in the simple form

$$y_p(t) = w'(t)\phi + h\exp[\Lambda t]\cdot x(0) \tag{7}$$

Pre-multiplying Eq. (7) by $w(t)$ and taking integral over the time period T, the least-squares estimate of ϕ is given as[13]

$$\hat{\phi}(k+1) = \left[\int_{kT}^{(k+1)T} w(t)w'(t)\mathrm{d}t\right]^{-1} \times \left[\int_{kT}^{(k+1)T} w(t)\{y_p(t) - h\exp(\Lambda t)\cdot\hat{x}(0)\}\mathrm{d}t\right] \tag{8}$$

where

$$\hat{\phi}(k+1) \triangleq [\hat{f}'(k+1),\ \hat{g}'(k+1)]' \tag{9}$$

is the estimate of ϕ at $t = (k+1)T$, and is determined from measurements for $kT \le t < (k+1)T$. $\hat{x}(0)$ is the initial estimate of the state at the designer's disposal. The inverse of $[\int_{kT}^{(k+1)T} w(t)w'(t)]$ exists if the input $u_p(t)$ is sufficiently rich in frequencies.[8] Substituting Eq. (7) into Eq. (8) gives

$$\hat{\phi}(k+1) = \phi\left[\int_{kT}^{(k+1)T} w(t)w'(t)\mathrm{d}t\right]^{-1} \times \left[\int_{kT}^{(k+1)T} w(t)\cdot h\exp(\Lambda t)\cdot(\hat{x}(0) - x(0))\mathrm{d}t\right] \tag{10}$$

On the other hand, $y_m(t)$ in Eq. (3) can be expressed in the same way as Eq. (7) using the estimated values $\hat{\phi}(\cdot)$ and $\hat{x}(0)$:

$$y_m(t) = w'(t)\hat{\phi}(k) + h\exp[\Lambda t]\cdot\hat{x}(0) \quad \text{for} \quad kT \le t < (k+1)T \tag{11}$$

where, $x_m(0) = \hat{x}(0)$. Defining the output error between $y_p(t)$ and $y_m(t)$ as

$$e(t) = y_m(t) - y_p(t) = w'(t)[\hat{\phi}(k) - \phi] + h\exp[\Lambda t]\cdot[\hat{x}(0) - x(0)] \tag{12}$$

and substituting Eq. (12) into Eq. (10), the updating algorithm to be sought becomes

$$\hat{\phi}(k+1) = \hat{\phi}(k) - \left[\int_{kT}^{(k+1)T} w(t)w'(t)\mathrm{d}t\right]^{-1} \cdot \left[\int_{kT}^{(k+1)T} w(t)e(t)\mathrm{d}t\right] \tag{13}$$

As can be seen from Eq. (10), $\hat{\phi}(k+1)$ converges exponentially to ϕ as $k \to \infty (t \to \infty)$. In addition, $x_m(t)$, the estimated value of the state vector $x(t)$, described by

$$x_m(t) = Z(t)\hat{f}(k) + V(t)\hat{g}(k) + \exp[\Lambda t]\cdot\hat{x}(0) \quad \text{for} \quad kT \le t < (k+1)T \tag{14}$$

also converges exponentially to $x(t)$ as $t \to \infty$. Hence, Eqs. (11, 13, and 14) define an adaptive observer and identifier whose schematic diagram is illustrated in Fig. 2.

It must be noted that this adaptive observer is similar in some respects to that described in Ref. 10, in that the observer dynamics are represented as an algebraic function of the filter states and the identified parameters, and the estimates of the state vector and unknown parameters converge to their true values exponentially. The identification algorithm of Eq. (13) does not require the selection of weighting parameters, which is unavoidable in the algorithms based on stability criteria.[8,9,11] Moreover, this observer does not require auxiliary signals for achieving global asymptotical convergence of the observation process. Such signals are characteristic of conventionally designed adaptive observers.[8,9] It is believed that the canonical form employed here simplifies the derivation of the observers discussed in Ref. 10.

Design of an Updated Optimal Controller

The design of the updated optimal controller is based here upon model Eq. (3) [or Eqs. (11) and (14)], which represents the current knowledge of the plant. Consider only the first-level control shown in Fig. 1 (that is, $u_p = u_m = u_0 + u_r$). The updated optimal controller is designed to produce an optimal input $u_m(t)$ so as to minimize a quadratic performance index J of the form

$$J = \frac{1}{2}\int_0^\infty [Qy_m^2(t) + Ru_m^2(t)]\mathrm{d}t \tag{15}$$

where Q and R are positive constants. The control law is given by

$$u_m(t) = -Lx_m(t) + u_r(t) \tag{16a}$$

$$L \triangleq R^{-1}\hat{g}'K \tag{16b}$$

where $n \times n$ positive definite symmetric matrix K satisfies the algebraic Riccati equation:

$$\hat{F}'K + K\hat{F} - K\hat{g}R^{-1}\hat{g}'K + h'Qh = 0 \tag{17}$$

According to Eqs. (16) and (17), the gain matrix L is updated every T seconds when new estimates of $\hat{F}$ and $\hat{g}$ are obtained. Since $(\hat{F},\hat{g})$ are not time-invariant, the same conditions as described in Ref. 7 are necessary for the derivation of the control law [Eq. (16)].

To solve Eq. (17) efficiently, the procedure suggested by Kleinman[14] is used. The iterative algorithm involves the following steps:

1) Selecting a matrix L_0 such that the matrix $A_0 = \hat{F} - \hat{g}L_0$ has eigenvalues with negative real parts.

2) Solving for V_k, the linear algebraic equation,

$$A_k'V_k + V_kA_k + h'Qh + L_k'RL_k = 0 \tag{18}$$

3) Computing

$$L_k = R^{-1}\hat{g}'V_{k-1} \tag{19a}$$

$$A_k = \hat{F} - \hat{g}L_k \tag{19b}$$

As $k \to \infty$ the foregoing procedure leads to the solution of Eq. (17).

Simulation Study

To evaluate the effectiveness of the system just described, a simulation study was carried out for the pitch attitude control of a STOL aircraft. The data that were used were taken from Ref. 15 and are those for the Dornier DO-28 D STOL "Sky Servant" aircraft.

The linearized longitudinal dynamics of the aircraft motion in the stability axis about the equilibrium point are given by the state equations

$$\dot{x}_p = A_px_p + b_pu_p \tag{20a}$$

$$y_p = c_px_p \tag{20b}$$

where

$$x_p = \begin{bmatrix} \theta \\ q \\ w \\ u \end{bmatrix} \begin{matrix} \text{(pitch attitude)} \\ \text{(pitch rate)} \\ \text{(vectical velocity)} \\ \text{(forward velocity)} \end{matrix}$$

$u_p = \delta e$ (elevator deflection angle)

$y_p = \theta$

Table 1 Definition of the flight condition[15]

No.	Flight conditions	Mass, kg	Altitude, m	Forward velocity, V_0, m/s	Flap position, deg	Glide path angle γ_0, deg	Dynamic pressure q_D, N/m^2
1	Cruise, fully loaded	3500	2000	77.8	0	0	3046
2	Cruise, without payload	2360	2000	77.8	0	0	3046
3	Descent	3400	0	33.4	20	−5.4	683
4	Steep descent	3500	0	30.9	52	−10.4	585

and the coefficient matrices in Eq. (20) are

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -g\sin\gamma_0 M_{\dot w} & M_q + (V_0 + Z_q)M_{\dot w} & M_w + Z_w M_{\dot w} & M_u + Z_u M_{\dot w} + (y_F m/I_y)F_u \\ -g\sin\gamma_0 & V_0 + Z_q & Z_w & Z_u \\ -g\cos\gamma_0 & 0 & X_w & X_u + F_u \end{bmatrix} \tag{21a}$$

$$b_p' = [0 \quad M_{\delta e} + Z_{\delta e} M_{\dot w} \quad Z_{\delta e} \quad x_{\delta e}] \tag{21b}$$

$$c_p = [1 \quad 0 \quad 0 \quad 0] \tag{21c}$$

Simulation studies were conducted for the four flight conditions (FC) shown in Table 1. However, in this paper results are presented only for the two extremes, **FC 1** and **FC 4.** For purposes of information, the values of the coefficient matrices A_p and b_p are given in Table 2, from which it can be seen that there are large variations in the dynamics of the aircraft between these two flight conditions.

Study of the Identification Scheme

To illustrate the effectiveness of the identification and estimation procedure, digital simulations were first made using the short-period approximation of Eq. (21). The approximated dynamics consist of the three-dimensional system less the contribution of the forward velocity. The adaptive observer that is employed, Eqs. (11) and (14), is also of this order, which corresponds to the approximation of the plant. The control switches in Fig. 1 were set to be in the off position (that is $u_p = u_m = u_r$) for the identification and estimation purpose. The poles of the filter in Eq. (4), which was used for generating the auxiliary signals, were selected to be at

$$\lambda_1 = 4.0 \qquad \lambda_2 = 3.0 \qquad \lambda_3 = 0.5 \tag{22}$$

and the updating period T of the parameters was selected to be 1s. A rectangular pulse of period 2 s was used as a command input u_r.

Figures 3a and 3b show the time histories of the identified values of the unknown parameters for **FC** 1 and **FC** 4, where all of the initial conditions are zero. These figures illustrate the rapid and stable character of the adaptive observer. Figure 3a shows better responses than Fig. 3b because the poles, $-\lambda_i$, of the filter were selected for **FC** 1 and not **FC** 4. These figures also show that the output error e between y_m and y_p for each flight condition was nearly equal to zero after about 4 s. The state variables $x_m(t)$ of the adaptive observer were seen to converge to their true values $x(t)$ at almost the same rate. The updating period T also affects these responses. At the expense of overshoot, the speed of convergence of the parameters to their true values can be made faster by making T smaller. However, to avoid excessive overshoot, a value of $T=2$ was selected in the following simulation studies.

Table 2 Coefficient matrices for flight conditions 1 and 4

FC No.	A_p				b_p
1	0	1	0	0	0
	0	−4.625	−0.136	−0.0147	−39.488
	0	75.896	−1.643	−0.2520	−20.710
	−9.800	0	0.0739	−0.0490	0
4	0	1	0	0	0
	−0.0283	−2.233	−0.0643	−0.0214	−7569
	1.769	29.979	−0.735	−0.636	−3.977
	−9.639	0	0.205	−0.143	0

Simulation Studies of Short-Period Dynamics of STOL Aircraft

The effectiveness of the two-level adaptive controller was examined using the same approximate dynamics of the plant and adaptive observer and the same command input u_r as used in the above example. The updated optimal controller was synthesized using the weighting coefficients $Q=1$, $R=10$ in the performance index [Eq. (15)]. The matrix Riccati Eq. (17) was solved using the recursive algorithm given by Eqs. (18) and (19). For each updating period T, five iterations were carried out so as to calculate the feedback gain matrix L, based upon the current status of the adaptive observer.

The error servo consists of a feedback signal for the pitch attitude angle error between the plant and the adaptive observer. The numerical value of its gain, which was determined using modal control theory, was taken from Ref. 15 for the purpose of a conventional servo system, and is

$$K_\theta = 0.12 + 0.38\, q_{D1}/q_{Di} \tag{23}$$

where q_{Di} denotes the dynamic pressure of the ith flight condition.

Results of the simulation studies are shown in Figs. 4 and 5 for the flight conditions 1 and 4, respectively. Five time histories are illustrated for each flight condition: the output of the plant y_p, the output error $e = y_m - y_p$, the control input to the plant u_p, the estimated values of parameters $\hat{f}$ and $\hat{g}$, and the feedback gain L. Optimal values with a priori knowledge of the plant's dynamics are also shown for purposes of comparison. These simulation studies were performed by initially having switches CS 1 and CS 2 opened until the adaptive model yielded the first estimate of the plant at $t=2$s. The initial conditions of the plant and the adaptive observer [Eq. (14)] were set equal to zero for each flight condition, and the initial values of the unkown parameters were assumed to be

$$\hat{f}(0) = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix} \qquad \hat{g}(0) = \begin{bmatrix} 10 \\ -10 \\ 2 \end{bmatrix}$$

As seen from Figs. 4a and 5a, the error e between the outputs of the plant and the adaptive observer is almost zero after the third adapatation at $t=6$ s. As shown by Figs. 4b and 5b, the proposed adaptive observer yields rapid estimations of the unknown parameters. The first-level and two-level adaptive controllers produce the same time histories of estimates values of unknown parameters and feedback gains. At $t=4$ s of the second estimation they are approximately equal to the optimum values. It is also to be noted that, owing to the additional control produced by the error servo at the second level, the output y_p of the two-level adaptive control scheme converges more rapidly to the optimal one than that of the first-level adaptive control scheme. Both of the

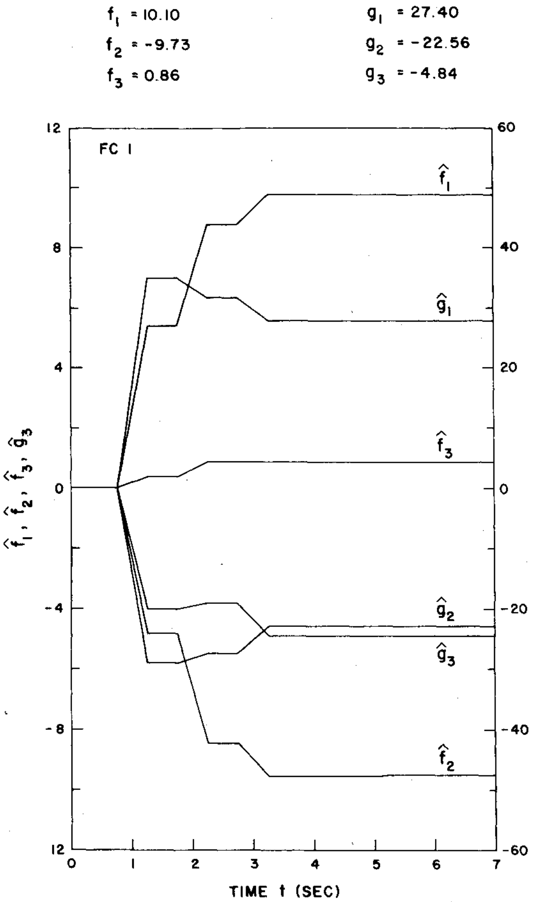

Fig. 3a Estimated values of the parameters for FC 1.

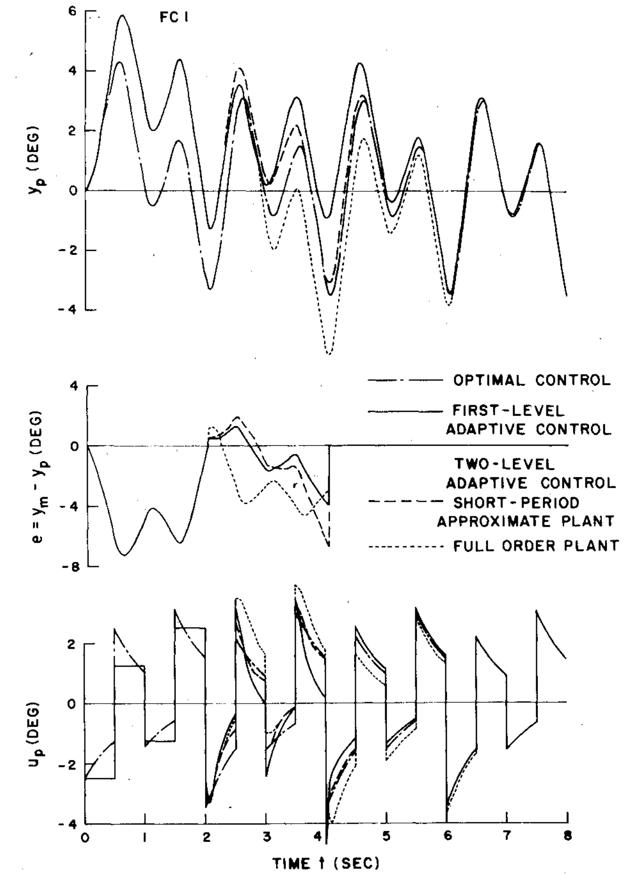

Fig. 4a Output y_p and input u_p of the plant and output error e for FC 1.

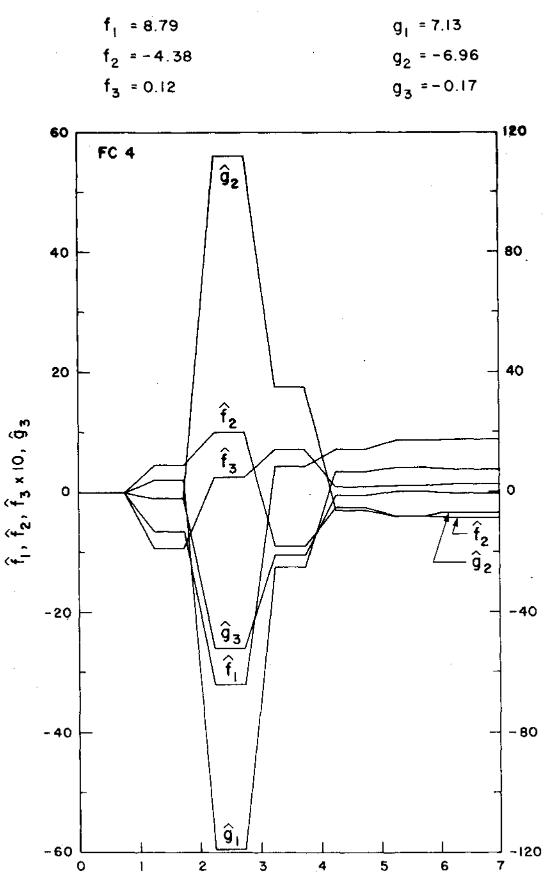

Fig. 3b Estimated values of the parameters for FC 4.

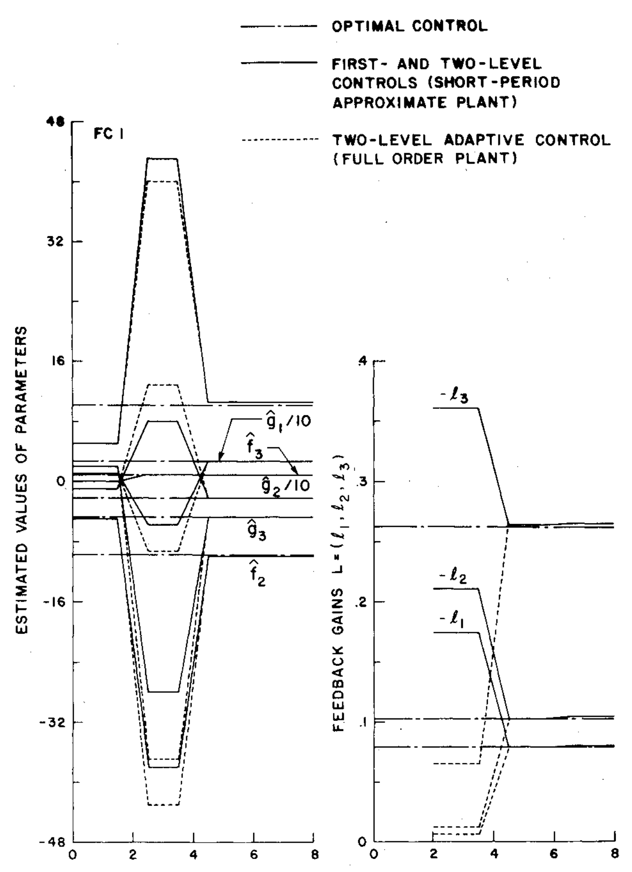

Fig. 4b Estimated values of parameters and feedback gain L for FC 1.

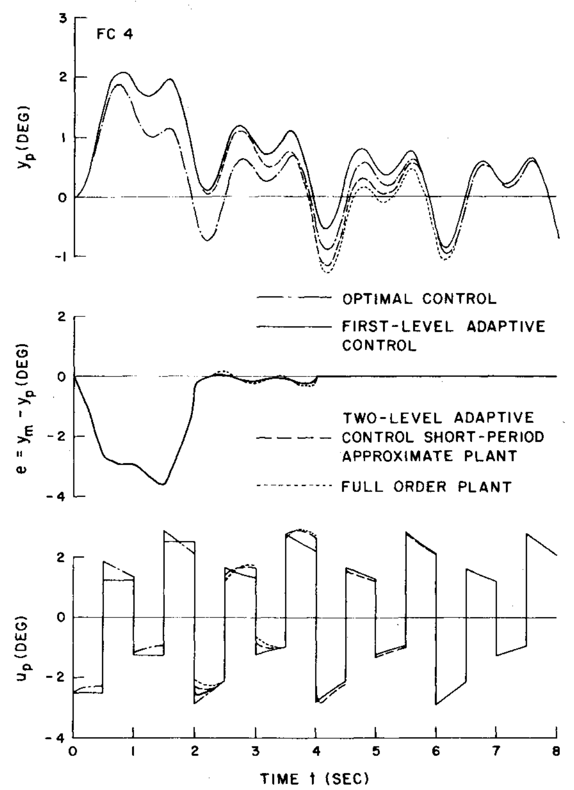

Fig. 5a The output y_p and input u_p of the plant and output error e for FC 4.

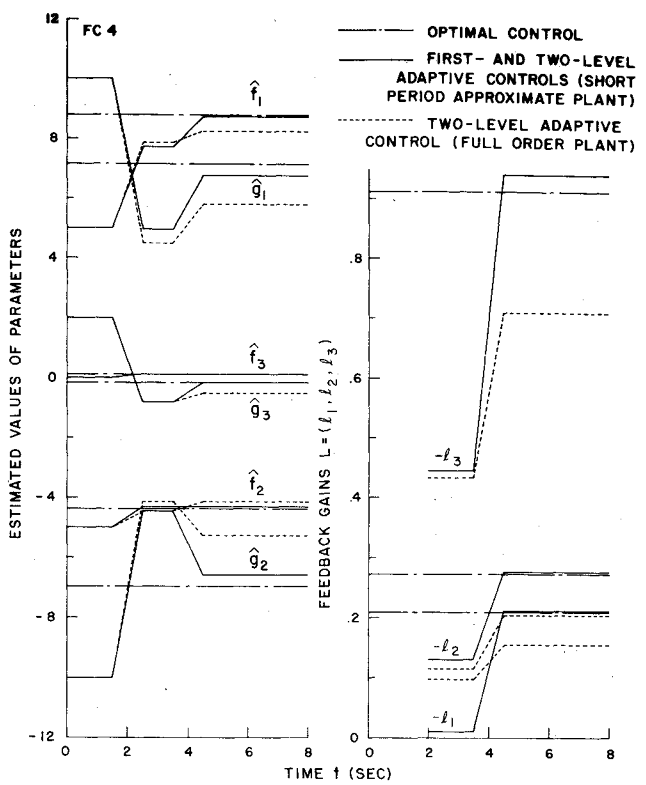

Fig. 5b Estimated values of the parameters and feedback gain L for FC 4.

schemes, however, produce almost optimal responses even when the adaptive observer does not yield a good estimate over the first estimation period.

Simulation Studies of Full-Order Dynamics of STOL Aircraft

In the next stage of the study, simulation studies were performed using the fourth-order dynamics of the STOL aircraft described in Eq. (21) and the previously described third-order adaptive observer; that is, the dimension of the observer is smaller than that of the plant. The numerical values of the parameters and initial conditions that were used here were the same as in the above example.

The time histories of the two-level adaptive control scheme using full-order dynamics of the plant are illustrated by the dotted lines in Fig. 4 for FC 1 and in Fig. 5 for FC 4. It is seen that the estimated values of the unknown parameters are almost the same as those obtained using the short-period dynamics. For FC 1 the output y_p, using the full-order dynamics, deviates from that using the short-period dynamics after the first adaptation. This is because the first estimates of $\hat{g}_1$ and $\hat{g}_2$ have greater overshoots, which lead to the poor estimates of the feedback gains. The output y_p converges, however, rapidly to the optimal value after the second adaptation. For FC 4, the output y_p, using full-order dynamics, also shows a good response.

Conclusion

In this paper, a two-level adaptive control scheme was developed for linear unknown plants. The unknown dynamics of the plant were identified and its state variables were estimated using an adaptive observer. Based upon this estimation, the feedback gains of an optimal controller were updated. This controller then provided a control signal at the first level. A conventional error servo provided the control at the second level. The adaptive observer was designed using a canonical form for modeling the unknown aircraft dynamics. The observer dynamics were represented as an algebraic function of certain filter states and have an exponential rate of convergence. The effectiveness of the proposed adaptive method was demonstrated using simulation studies of the pitch attitude control system of a STOL aircraft under two different flight conditions. Results of the simulation studies for short-period dynamics, as well as for full-order dynamics, show a rapid rate of convergence of the adaptive observer, and compare favorably with those obtained using an optimal controller with a priori knowledge of the dynamics of the plant.

Appendix: Derivation of the Canonical Form in Eq. (2)

Consider an n dimensional single output completely observable system (A, c) which is represented by the standard observable canonical form:

$$A = \left[a \;\middle|\; \begin{matrix} 1 & \cdots & 0 \\ & \ddots & \\ 0 & \cdots & 1 \\ 0 & \cdots & 0 \end{matrix} \right] \qquad a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \tag{A1}$$

$$c = [\,1 \mid 0 \cdots 0\,]$$

If a single-output system is completely observable, it can be transformed into the canonical form of Eq. (A1) by using the procedure given by Lüders and Narendra.[16]

The problem here is to find a nonsingular matrix T which transforms the pair (A,c) into the canonical form given by Eq. (2), that is,

$$F = TAT^{-1} \qquad h = cT^{-1} \tag{A2}$$

where

$$F = \Lambda + [f\!:\ldots:\!f] \qquad h = [1\!:\ldots:\!1]$$

$$\Lambda = \begin{bmatrix} -\lambda_1 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & -\lambda_n \end{bmatrix} \qquad f = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix} \qquad (\lambda_i \neq \lambda_j \text{ for } i \leq \neq j)$$

Let the following vectors be defined as:

$c[\Lambda]\epsilon R^{n+1} =$ a vector formed by the coefficients of the characteristic polynomial of $\prod_{k=1}^{n}(s+\lambda_k)$

$c(\Lambda/\lambda_i)\epsilon R^{n} =$ $(i=1,...,n)$ same as preceding, but the factor $(s+\lambda_i)$ is delected from the characteristic polynomial

A property of these vectors is

$$\begin{bmatrix} c(\Lambda/\lambda_i) \\ \hdashline 0 \end{bmatrix} + \lambda_i \begin{bmatrix} 0 \\ \hdashline c(\Lambda/\lambda_i) \end{bmatrix} = c(\Lambda) \qquad (i=1,...,n) \tag{A3}$$

Since the system matrices A and F must have the same characteristic polynomial, the two vectors $\boldsymbol{a}$ and $\boldsymbol{f}$ must satisfy the relation

$$\begin{bmatrix} 1 \\ \hdashline -\boldsymbol{a} \end{bmatrix} = c(\Lambda) - \sum_{i=1}^{n} f_i \begin{bmatrix} 0 \\ \hdashline c(\Lambda/\lambda_i) \end{bmatrix} \tag{A4}$$

The transformation matrix T in Eq. (A2) is defined to be

$$T^{-1} = [c(\Lambda/\lambda_1) : c(\Lambda/\lambda_2) : \ldots : c(\Lambda/\lambda_n)] \tag{A5}$$

By carrying out the matrix multiplication $AT^{-1} = T^{-1}F$, and by using Eqs. (A3) and (A4), both sides of the equation can be shown to be identical. Further, $cT^{-1} = h$. Also, since the column vectors of T^{-1} are all linearly independent, because they are generated by the set of numers $\lambda_i (i=1,...,n)$ which are all distinct, T is a nonsingular matrix.

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